2022

# AP® Physics C: Electricity and Magnetism

## Scoring Guidelines
## Set 2

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## Question 1: Free-Response Question — 15 points

**(a)** For correctly determining the charge on the outer surface of the shell — **1 point**

**Example Response**

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +4Q - (-Q)$$
$$q_{outer} = +5Q$$

**Scoring Note:** A correct response may earn a point even if no work is shown.

**Total for part (a) — 1 point**

**(b)** For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

**Example Response**

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E(2\pi rL)$$

For using a correct expression for $q_{enc}$ as a function of $r$ — **1 point**

**Example Response**

$$q_{enc} = \rho V = \left(\frac{Q}{\pi R^2 L}\right)\left(\pi r^2 L\right) = Q\frac{r^2}{R^2}$$

For using the correct area — **1 point**

**Example Response**

$$E = q_{enc}\frac{1}{\varepsilon_0(2\pi rL)} = \left(Q\frac{r^2}{R^2}\right)\frac{1}{\varepsilon_0(2\pi rL)}$$
$$E = Q\frac{r}{2\pi\varepsilon_0 R^2 L}$$

**Total for part (b) — 3 points**

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| **(c)** | For recognizing that the $E$-field varies as the reciprocal of the separation distance from the center of the nonconducting cylinder<br><br>**Example Response**<br><br>$E \propto \frac{1}{r}$ | **1 point** |
| | For the correct value, including units, of electric field at $2R$<br><br>**Example Response**<br><br>$E_{new} = \frac{E_{old}}{2} = \frac{12 \text{ N/C}}{2}$<br>$E_{new} = 6 \text{ N/C}$ | **1 point** |
| | **Total for part (c)** | **2 points** |
| **(d)** | For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral<br><br>**Example Response**<br><br>$V_f - V_i = -\int_{s_i}^{s_f} E dr$<br>$\Delta V = -\int_{r=R}^{r=3R} E dr$ | **1 point** |
| | For substituting the correct expression for the electric field or an expression consistent with the explicit functional dependence of $E(r)$ from part (c)<br><br>**Example Response**<br><br>$\Delta V = -\int_{r=R}^{r=3R} \left( \frac{Q}{2\pi\varepsilon_0 r L} \right) dr = -\frac{Q}{2\pi\varepsilon_0 L} \int_{r=R}^{r=3R} \frac{1}{r} dr$ | **1 point** |
| | For correctly integrating the electric field expression that was substituted in the previous point with the correct limits of of integration<br><br>**Example Response**<br><br>$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L} [\ln(r)]_{r=R}^{r=3R} = -\frac{Q}{2\pi\varepsilon_0 L} [\ln(3R) - \ln(R)]$<br>$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L} \ln(3)$ | **1 point** |
| | **Total for part (d)** | **3 points** |

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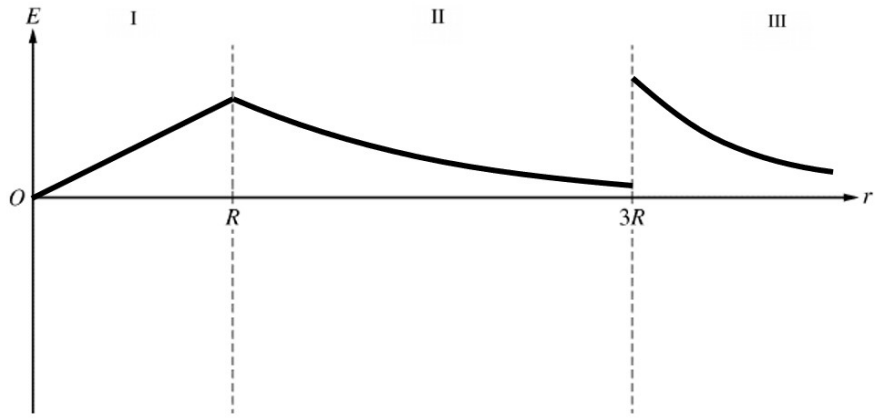
| **(e)(i)** | For indicating $E$ is positive and the magnitude increases linearly from 0 to $R$ | **1 point** |
| | For a positive, decreasing concave up graph for $r > R$ | **1 point** |
| | For a discontinuous increase in magnitude at $r = 3R$ | **1 point** |

**Example Response**

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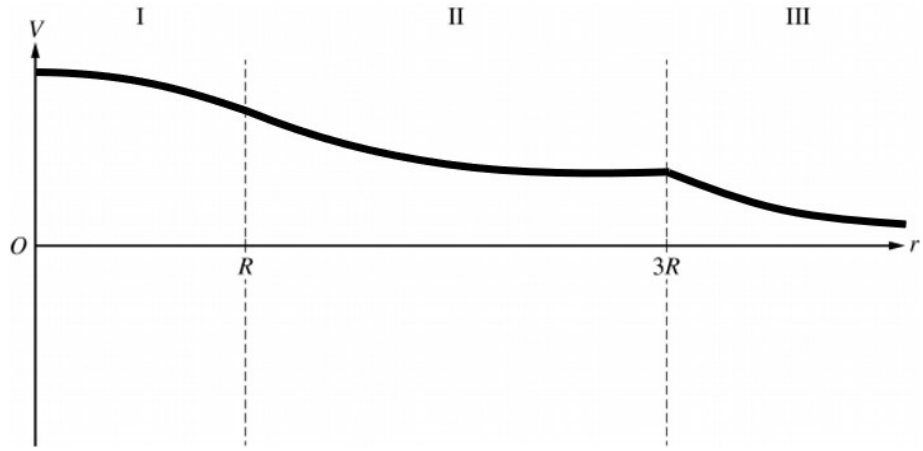
| **(e)(ii)** | For a graph that is concave down and always decreasing as $r$ increases within Region I | **1 point** |
| | For a graph that is concave up and always decreasing as $r$ increases within each of the Regions II and III | **1 point** |
| | For a continuous graph across all three regions | **1 point** |

**Scoring Notes:**

- The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.
- The response can earn a maximum of 2 points if all three segments are reflected across the horizontal axis.

**Example Response**




**Total for part (e) 6 points**

**Total for question 1 15 points**

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## Question 2: Free-Response Question — 15 points

**(a)** For a schematic diagram with the capacitor in series with the resistor — **1 point**

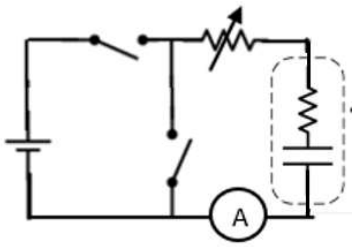
**Scoring Note:** The response may earn this point even if the variable resistor is not included in the circuit diagram.

For a schematic diagram with the ammeter in series with the capacitor and resistor — **1 point**

For a schematic diagram that uses a switch to connect the battery to the capacitor — **1 point**

For a schematic diagram that uses a switch that allows the capacitor to discharge through the resistor — **1 point**

**Example Response**




**Total for part (a) — 4 points**

**(b)** For using an appropriate loop equation by substituting a correct expression for the potential difference across the capacitor in terms of $I$, $C$, and $r_C$ and $IR$ for the potential difference across the variable resistor, if included — **1 point**

**Example Response**

$$V_C - V_R = 0$$

$$\frac{q}{C} - Ir_C = IR \therefore \frac{q}{C} = I(R + r_C)$$

For substituting $R + r_C$ as the total resistance of the circuit — **1 point**

**Example Response**

$$R_{\text{total}} = R + r_C$$

**Scoring Notes:**

- This point is earned if the above substitution is made anywhere in part (b).
- If the variable resistor is not included in the expression, accept expressions without $R$ throughout.

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For a correct differential equation consistent with the first point that could be used to determine the current $I$ through the capacitor as a function of time $t$ **1 point**

**Example Response**

$$\frac{dq}{dt}\frac{1}{C} = (R + r_C)\frac{dI}{dt}$$
$$-I\frac{1}{C} = (R + r_C)\frac{dI}{dt}; \quad I = -\frac{dq}{dt}$$

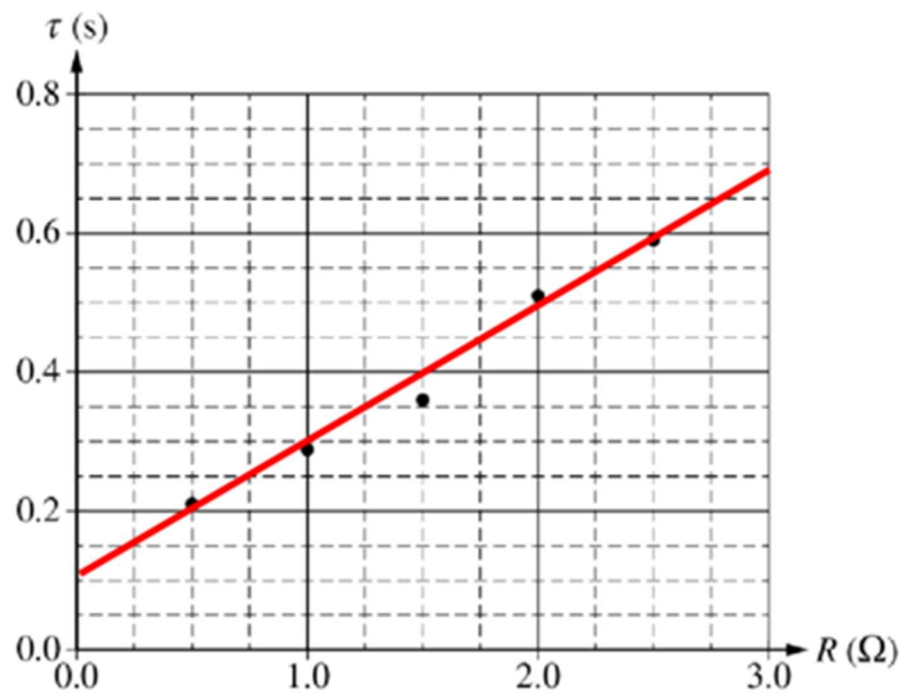
Where $q$ is the charge on one plate of the capacitor that decreases over time

$$\frac{1}{I}\frac{dI}{dt} = \frac{-1}{C(R + r_C)}$$
$$\ln\frac{I}{I_0} = \frac{-t}{C(R + r_C)}$$
$$I = I_0 e^{\frac{-t}{C(R+r_C)}}$$

**Total for part (b) 3 points**

**(c)(i)** For drawing an appropriate best-fit line **1 point**

**Example Response**

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| | | |
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| **(c)(ii)** | For correctly determining the slope of the line | **1 point** |

**Example Response**

$$\text{slope} = \frac{(0.24 - 0.08)\ \text{s}}{(1.0 - 0.2)\ \Omega} = 0.2\ \text{F}$$

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| | For using the equation $\tau = RC$ to determine that the slope must be $C$ | **1 point** |

**Example Response**

$\tau = RC$

$\tau = (R + r_C)C$

$\tau = CR + r_C C$

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| | For correctly relating the vertical intercept to the internal resistance of the capacitor | **1 point** |

**Example Response**

$\text{Vertical intercept} = \tau_0 = r_C C$

$\text{Vertical intercept} = 0.08\ \text{s}$

$$r_C = \frac{\text{vertical intercept}}{C}$$

$$= \frac{0.08\ \text{s}}{0.2\ \text{F}}$$

$r_C = 0.4\ \Omega$

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| | **Total for part (c)** | **4 points** |
| **(d)** | For selecting "Less than" and an attempt at a relevant justification | **1 point** |
| | For a correct justification that the internal resistance would be less due to the unknown resistance that is measured being the equivalent resistance of the ammeter and capacitor | **1 point** |

**Example Response**

*The internal resistance of the ammeter would add to the internal resistance of the capacitor due to the fact the circuit elements are in series; this would result in an equivalent resistance that is measured in this experiment. Thus, the internal resistance of the capacitor is smaller than this equivalent resistance measured.*

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| | **Total for part (d)** | **2 points** |
| **(e)** | For selecting "Remain unchanged" and an attempt at a relevant justification | **1 point** |
| | For a correct justification that the slope of the line is capacitance, which is independent of resistance | **1 point** |

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**Example Response**

*The relationship between the time constant and the resistance, the slope, is the capacitance, which does not change regardless of how large the value of the resistance is.*

**Scoring Note:** This point is scored with consistency with the circuit drawn in part (a).

**Total for part (e) 2 points**

**Total for question 2 15 points**

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## Question 3: Free-Response Question — 15 points

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| **(a)** | For selecting "Counterclockwise" and an attempt at a relevant justification | **1 point** |
| | For a justification that correctly relates how the changing current in the solenoid changes the flux through the loop with respect to time | **1 point** |
| | For a justification that has a correct relationship between the change in magnetic flux through the loop of wire and how the current in the loop changes to oppose the change in magnetic flux | **1 point** |
| | **Example Response**<br>*In the end view, the magnetic field due to the solenoid is directed into the page. The current in the solenoid is increasing; thus, the magnetic flux is increasing. According to Lenz's law, because the magnetic flux is increasing and into the page, the current in the loop must create a magnetic field directed out of the page; thus, the current in the loop must be counterclockwise the counter the change in magnetic flux.* | |
| | **Total for part (a)** | **3 points** |
| **(b)** | For using an appropriate equation to calculate emf in the loop<br>**Example Response**<br>$\varepsilon = \left\lvert \frac{d\Phi}{dt} \right\rvert = \frac{d(BA)}{dt} = A\frac{dB}{dt}$ | **1 point** |
| | For correctly substituting the magnetic field for a solenoid into above equation<br>**Example Response**<br>$\varepsilon = A\frac{d(\mu_0 nI)}{dt} = \mu_0 nA\frac{dI}{dt}$ | **1 point** |
| | For using Ohm's law to calculate the current in the loop<br>**Example Response**<br>$I_{\text{loop}} = \frac{\varepsilon}{R} = \frac{\mu_0 nA}{R}\frac{dI}{dt}$ | **1 point** |
| | For correctly substituting into the previous equation for the induced current in the loop<br>**Example Response**<br>$I_{\text{loop}} = \frac{\left(4\pi \times 10^{-7}\ \text{T•m/A}\right)\left(\frac{500\ \text{turns}}{0.25\ \text{m}}\right)\left((\pi)(0.10\ \text{m})^2\right)}{(3\ \Omega)}(5.0\ \text{A/s}) = 1.3 \times 10^{-4}\ \text{A}$ | **1 point** |
| | **Total for part (b)** | **4 points** |

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| **(c)** | For using an appropriate expression for the power for a constant current through a resistor<br><br>**Example Response**<br>$P = I_{\text{loop}}{}^2 R$ | **1 point** |
| | For a correct substitution consistent with (b) into a correct expression for electrical power<br><br>**Example Response**<br>$P = I_{\text{loop}}{}^2 R = \left(\frac{\mu_0 nA}{R}\frac{dI}{dt}\right)^2 R = \left(1.3 \times 10^{-4}\ \text{A}\right)^2 (3.0\ \Omega) = 5.2 \times 10^{-8}\ \text{W}$ | **1 point** |
| | For correctly substituting the power dissipated by the loop of wire into a correct energy equation<br><br>**Example Response**<br>$P = \frac{dE}{dt} = \frac{\Delta E}{\Delta t} \therefore \Delta E = P\Delta t$<br>$\Delta E = \left(5.2 \times 10^{-8} W\right)(2.0\ \text{s}) = 1.0 \times 10^{-7}\ \text{J}$ | **1 point** |
| | **Total for part (c)** | **3 points** |
| **(d)** | For selecting "The plane of the loop is not perpendicular to the axis of the solenoid." | **1 point** |
| | For correctly justifying the selection<br><br>**Scoring Note:** A response cannot earn this point if the incorrect selection is chosen.<br><br>**Example Response**<br>*If the plane of the loop is not perpendicular to the axis of the solenoid it is not perpendicular to the magnetic field. Therefore, the magnetic flux and emf will be less, so the current will be less.* | **1 point** |
| | **Total for part (d)** | **2 points** |
| **(e)** | For selecting "$\frac{I_2}{I_1} = 2$" with an attempt at a relevant justification<br><br>**Scoring Note:** The response can earn this point if selecting "$\frac{I_2}{I_1} > 2$" and only referencing the increase in flux through the loop in the justification. | **1 point** |
| | For a statement that indicates that the resistance of the second loop is double the resistance of the original loop | **1 point** |
| | For a statement that indicates that the new emf is four times greater than the original emf due to the increased flux through the loop resulting from the quadrupled area | **1 point** |

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**Example Response**

*The magnetic flux quadruples. This is because of the increase in area of the loop, which quadruples the emf. The resistance of the loop doubles because the length of the wire doubles. Therefore,* $\frac{I_2}{I_1} = 2$.

$$\varepsilon = \mu_0 n\left(A_{loop}\right) = \mu_0 n\left(\pi r^2\right)\frac{dI}{dt}$$

*The radius of the loop doubles. Therefore, the area of the loop quadruples.*

$$R = \rho\frac{2\pi r}{A_{wire}}$$

*The radius of the loop doubles. Therefore, the resistance of the loop doubles.*

$$\frac{I_2 = \frac{\varepsilon}{R} = \frac{\mu_0 n(4)A}{(2)R}\frac{dI}{dt}}{I_1 = \frac{\varepsilon}{R} = \frac{\mu_0 nA}{R}\frac{dI}{dt}} = 2$$

**Total for part (e) 3 points**

**Total for question 3 15 points**

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